computed filter characteristics in Fig. 4 were obtained by assuming that all the elements of the triplexer have the same unloaded Q value as a stripline of 50 Ω in a Rexolite 2200 circuit board. No final adjustment of the dimensions was required to obtain this response. The resonator length reduction factor was 2.50 percent of a quarter of a wavelength at 3.0 GHz, as was calculated by Lagerlöf [7]. The electrical length of the corner was measured and found to be 9.62 percent of a quarter wavelength at the same frequency. The alignment of the center frequencies of the filters was of great importance. Care had to be taken in the photoetch process to get a filter requiring no final adjustment. Insertion loss at frequencies outside the crossover region was very low due to the loose coupling of the bandstop filter resonators.

References

[1] G. Matthaei and E. Cristal, "Multiplexer channel-separating units using interdigital and parallel-coupled filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 328–334, May 1965.
[2] R. Wenzel, "Printed-circuit complementary filters for narrow bandwidth multiplexers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 147–157, Mar. 1968.
[3] R. Kihlén, "Filterbank for microwave frequency" (in Swedish), Chalmers Univ. of Technology, Gothenburg, Sweden, Internal Rep. 6905, Dec. 1969.
[4] R. Wenzel, "Theoretical and practical applications of capacitance matrix transformations to TEM network design," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-14, pp. 635–647, Dec. 1966.
[5] S. B. Cohn, "Shielded coupled-strip transmission line," *IRE Trans. Microwave Theory Tech.*, vol. MTT-3, pp. 29–38, Oct. 1955.
[6] W. J. Getsinger, "Coupled rectangular bars between parallel plates," *IRE Trans. Microwave Theory Tech.*, vol. MTT-10, pp. 65–72, Jan. 1962.
[7] R. Lagerlöf, "Foreshortening of half-wave stripline resonators," Chalmers Univ. of Technology, Gothenburg, Sweden, Tech. Rep. 7106, Dec. 1971.

Implementation of Conservation-of-Energy Condition in Small Aperture and Small Obstacle Theory

CHUNG-LI REN

Abstract—In a previous paper, Felsen and Kahn showed that the scattering matrix of small apertures and obstacles in multimode waveguide regions is conveniently calculated for general lossless structures, but observed that the scattering matrix does not satisfy the conservation-of-energy requirement. It is also to be noted that the scattering parameters could become much larger than unity or even infinite for frequencies near or at the cutoff of the coupled modes. A method is presented in this correspondence to implement the lossless condition so that the resultant scattering matrix satisfies the conservation-of-energy requirement and, consequently, can be represented as a lossless equivalent circuit for all frequencies. The corresponding impedance, admittance, and transfer matrices for general lossless symmetrical structures are given in compact form directly in terms of the scattering parameters.

I. Introduction

The design of waveguide components requires the availability of specific discontinuity structures with known transmission and reflection properties. A rigorous theoretical analysis of these waveguide discontinuities is very often quite involved and, in practice, its solution usually becomes tractable only with the imposition of judicious assumptions. One such assumption is that the apertures and the obstacles are small and the solutions may be evaluated easily in the lowest order of approximation, which is generally known as the small aperture and small obstacle theory [1]. The application of small aperture and small obstacle theory to discontinuities in multimode waveguide regions becomes particularly attractive in view of the fact that such design information is generally unavailable in the literature, whether in the form of theoretical calculation or measurements. The design of millimeter wave waveguide components, such as filters and couplers involving multimode propagation, is such an example.

However, the scattering matrix of a lossless waveguide discontinuity calculated from small aperture and small obstacle theory is not unitary and, hence, in violation of the conservation of energy [1]. Thus the impedance or admittance matrices, when converted from the scattering matrix, always contain real parts. In addition, the scattering parameters become very large or even infinite for frequencies at which certain modes are near or at cutoff. Therefore, meaningful equivalent circuits cannot be derived from these scattering matrices. In this correspondence, a technique is proposed to implement the conservation-of-energy condition so that the modified scattering matrices satisfy this condition. The corresponding impedance, admittance, and transfer matrices are derived in compact form directly in terms of the scattering parameters.

Manuscript received September 29, 1971; revised December 17, 1971.
The author is with Bell Telephone Laboratories, Incorporated, North Andover, Mass. 01845.

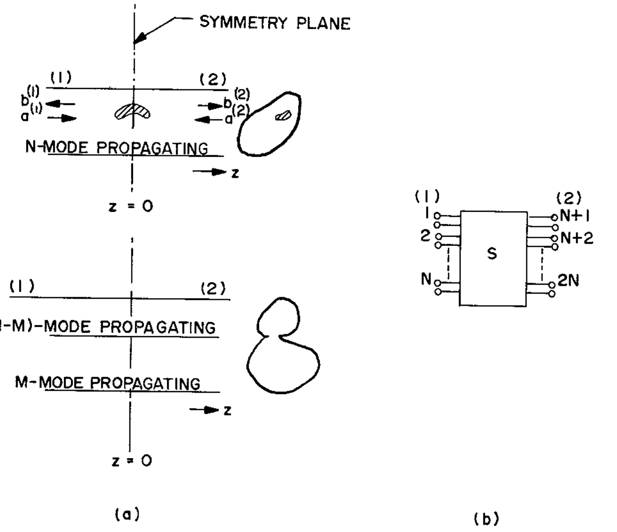

Fig. 1.

II. Small Aperture and Small Obstacle Scattering Formulation of Lossless Symmetrical Discontinuities in Multimode Waveguides

Consider the configuration in Fig. 1 where either a perfectly conducting obstacle or an aperture is located in a waveguide or between several waveguides propagating N modes. For convenience in the derivation of the theory, the structures of Fig. 1 are assumed to be symmetric in the sense that there exists a transverse plane of bisection at $z=0$. Such structures are the most frequently encountered discontinuities in the waveguide component designs.

The small aperture and small obstacle formulation for the scattering coefficients of the structures in Fig. 1 is given in equations (10) and (33) of [1], which may be generalized and written in a matrix form shown in (2).

$$b = Sa \qquad a = \begin{pmatrix} a^{(1)} \\ \vdots \\ a^{(2)} \end{pmatrix} \qquad b = \begin{pmatrix} b^{(1)} \\ \vdots \\ b^{(2)} \end{pmatrix} \tag{1}$$

$$S = \left(\begin{array}{c|c} A_1 + A_2 & I + A_1 - A_2 \\ \hline I + A_1 - A_2 & A_1 + A_2 \end{array} \right)_{2N \times 2N} \tag{2}$$

where I is the $N \times N$ identity matrix. A_1 and A_2 are imaginary $N \times N$ submatrices, which are functions of the polarizabilities of the discontinuity and the electromagnetic fields of the modes that are coupled by the discontinuity. In the general case, both A_1 and A_2 are nonzero. However, either A_2 or A_1 is a zero matrix for structures that are either pure shunt or pure series, respectively. For example, when all modes are coupled only through their longitudinal magnetic field components and (or) their transverse electric field components, A_1 is nonzero, $A_2=0$, and the structure is pure shunt. In the dual case when only the longitudinal electric field and (or) the transverse magnetic fields are coupled, A_2 is nonzero and $A_1=0$. The structure becomes pure series. It is to be noted that the scattering matrix in (2) is not unitary and does not satisfy the conservation-of-energy requirement [1]. In the submatrices A_1 and A_2, certain elements may become very large as frequencies move closer to the cutoff of certain modes.

III. Implementation of Conservation-of-Energy Condition via Z and Y Matrices in Terms of Small Aperture and Small Obstacle Scattering Parameters

It is shown [2] that the scattering matrix of (2) may be bisected into two parts as in (3). S^e represents the response of symmetrical excitation and S^o the antisymmetrical excitation.

$$S = S^e + S^o \tag{3}$$

where

$$S^{e}_{o} = \frac{1}{2}\left(\begin{array}{c|c} s^{e}_{o} & \pm s^{e}_{o} \\ \hline \pm s^{e}_{o} & s^{e}_{o} \end{array}\right)_{2N\times 2N}. \tag{4a}$$

The $N\times N$ even and odd submatrices s^e and s^o can be written in terms of A_1 and A_2 directly from (2) [2]:

$$s^e = I + 2A_1 \qquad s^o = -I + 2A_2. \tag{4b}$$

The corresponding impedance and admittance matrices are related to (4) by [2]

$$Z = \frac{1}{2}\left(\begin{array}{c|c} z^e + z^o & z^e - z^o \\ \hline z^e - z^o & z^e + z^o \end{array}\right)_{2N\times 2N} \qquad Y = \frac{1}{2}\left(\begin{array}{c|c} y^e + y^o & y^e - y^o \\ \hline y^e - y^o & y^e + y^o \end{array}\right)_{2N\times 2N} \tag{5}$$

$$z^{e}_{o} = (y^{e}_{o})^{-1} = (I + s^{e}_{o})(I - s^{e}_{o})^{-1}. \tag{6}$$

Substituting (4b) into (6),

$$z^e = (I + A_1)(-A_1)^{-1} \tag{7}$$

$$z^o = A_2(I - A_2)^{-1}. \tag{8}$$

As we can see from (7) and (8), z^e and z^o are not purely imaginary, but contain real parts. Since the discontinuities are assumed small, the magnitude of the elements of A_1 and A_2 is much smaller than unity; therefore, only the lowest order of approximation should be retained in (7) and (8), viz.:

$$z^e \simeq -A_1^{-1} \qquad y^e \simeq -A_1 \tag{9}$$

$$z^o \simeq A_2 \qquad y^o \simeq A_2^{-1}. \tag{10}$$

If these expressions are used, Z and Y of (5) are purely imaginary and completely define the lossless $2N$-port equivalent circuit of Fig. 1.

Therefore, the imaginary submatrices A_1 and A_2 of the scattering matrix from small aperture and small obstacle theory are substituted directly into (9) and (10) to obtain the equivalent circuit that fulfills the conservation-of-energy requirements retaining only the first order terms of A_1 and A_2.

IV. Modified Scattering Matrix for Small Aperture and Small Obstacle Theory

The scattering matrix can now be reconstructed in modified form using (6), (9), and (10). Thus we obtain

$$\hat{s}^e = (I - A_1)^{-1}(I + A_1) \tag{11}$$

$$\hat{s}^o = (A_2 + I)^{-1}(A_2 - I). \tag{12}$$

$\hat{s}^e$ and $\hat{s}^o$ satisfy $\hat{s}^e\hat{s}^{e*} = I$ and $\hat{s}^o\hat{s}^{o*} = I$, which are part of the conservation-of-energy condition [2].

The total scattering matrix with the lossless condition implemented can be obtained by substituting (11) and (12) into (3) and (4):

$$\begin{aligned} S &= S^e + S^o \\ &= \frac{1}{2}\left(\begin{array}{c|c} (I - A_1)^{-1}(I + A_1) & (I - A_1)^{-1}(I + A_1) \\ \hline (I - A_1)^{-1}(I + A_1) & (I - A_1)^{-1}(I + A_1) \end{array}\right) \\ &\quad + \frac{1}{2}\left(\begin{array}{c|c} (A_2 + I)^{-1}(A_2 - I) & -(A_2 + I)^{-1}(A_2 - I) \\ \hline -(A_2 + I)^{-1}(A_2 - I) & (A_2 + I)^{-1}(A_2 - I) \end{array}\right). \end{aligned} \tag{13}$$

From the comparison of (13) and (4) it becomes clear that the implementation of the lossless condition has added the higher order terms

$$\hat{s}^e - s^e = -2(-I + A_1)^{-1}A_1^2$$

$$\hat{s}^o - s^o = -2(I + A_2)^{-1}A_2^2$$

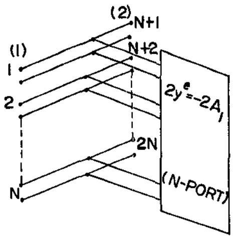

Fig. 2.

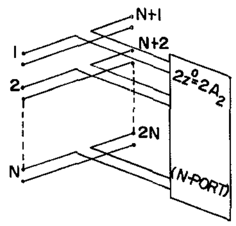

Fig. 3.

to the scattering matrix of (4). As a result, the modified scattering matrix in (13) can be represented by a lossless equivalent circuit for all values of A_1 and A_2 since S of (13) is always unitary, even as A_1 and A_2 become very large or infinite when certain modes are near or at the cutoff.

V. Pure Shunt and Pure Series Application

For practical applications, pure shunt and pure series cases are of special interest. A typical example of the pure shunt application is the directional coupler in which an array of small apertures is aligned in the longitudinal direction on the common waveguide wall and only the longitudinal magnetic field components are coupled through these apertures, $A_2 = 0$. The scattering matrix is therefore

$$S = \left(\begin{array}{c|c} (I - A_1)^{-1}A_1 & (I - A_1)^{-1} \\ \hline (I - A_1)^{-1} & (I - A_1)^{-1}A_1 \end{array}\right). \tag{14}$$

The equivalent circuit of (14) can be derived from the corresponding partial even admittance and odd impedance matrices of (9) and (10), respectively; $y^e = -A_1$ and $z^o = 0$.

$$Z = \frac{-1}{2}\left(\begin{array}{c|c} A_1^{-1} & A_1^{-1} \\ \hline A_1^{-1} & A_1^{-1} \end{array}\right). \tag{15}$$

The resultant equivalent circuit is shown in Fig. 2.

It must be emphasized that A_1 can be singular and the equivalent circuit of (15) can still exist because it is determined by the N-port partial network $y^e = -A_1$ (see Fig. 2). In the coupler design, either the transfer scattering matrix T or the transfer matrix of standing wave parameters $[ABCD]$ for each individual aperture may be used to obtain the total cascade equivalent circuit:

$$T = \left(\begin{array}{c|c} I + A_1 & A_1 \\ \hline -A_1 & I - A_1 \end{array}\right) \tag{16}$$

and

$$[A\ B\ C\ D] = \left(\begin{array}{c|c} I & 0 \\ \hline -2A_1 & I \end{array}\right). \tag{17}$$

These are derived directly in terms of A_1 and fulfill the conservation-of-energy condition.

The dual case is a pure series structure where, for example, only transverse magnetic field components are coupled through the apertures and $A_1 = 0$. The scattering matrix is

$$S = \left(\begin{array}{c|c} (A_2 + I)^{-1}A_2 & (A_2 + I)^{-1} \\ \hline (A_2 + I)^{-1} & (A_2 + I)^{-1}A_2 \end{array}\right). \tag{18}$$

The equivalent circuit of (17) can be derived from the corresponding partial even admittance and odd impedance matrices of (9) and (10), respectively; $y^e = 0$ and $z^o = A_2$. The equivalent circuit is shown in Fig. 3:

$$Y = \frac{1}{2}\left(\begin{array}{c|c} A_2^{-1} & -A_2^{-1} \\ \hline -A_2^{-1} & A_2^{-1} \end{array}\right). \quad (19)$$

The equivalent circuit of (19) exists even if A_2 is singular. The transfer scattering matrix and $[A\ B\ C\ D]$ matrix are

$$T = \left(\begin{array}{c|c} I - A_2 & A_2 \\ \hline -A_2 & I + A_2 \end{array}\right) \quad (20)$$

and

$$[A\ B\ C\ D] = \left(\begin{array}{c|c} I & 2A_2 \\ \hline 0 & I \end{array}\right) \quad (21)$$

respectively.

References

[1] L. B. Felsen and W. K. Kahn, "Network properties of discontinuities in multimode circular waveguide," *Proc. Inst. Elec. Eng.*, Feb. 1962.

[2] C. L. Ren, "Network representation for lossless symmetrical discontinuities in a multimode waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-14, pp. 81–85, Feb. 1966.

Computer Program Descriptions

Permeability Tensor of Magnetized Ferrites from Waveguide Measurements

PURPOSE: By means of this program the complex eigenvalues of the permeability tensor of magnetized ferrites can be calculated from measurements of the propagation constants of right- and left-handed rotating HE_{11} waves in circular cylindrical waveguides containing axial longitudinally magnetized ferrite rods.

LANGUAGE: Fortran IV; source program deck length 342 cards.

AUTHOR: H. Entschladen, Institut für Hoch- und Höchstfrequenztechnik der Ruhr-Universität Bochum, 4630 Bochum, Germany.

AVAILABILITY: ASIS-NAPS Document No. NAPS-01818.

DESCRIPTION: The structure of a circular cylindrical waveguide containing an axial longitudinally magnetized ferrite rod allows the measurement of the microwave material parameters of ferrites, i.e., the permeability tensor, by using fairly large ferrite specimens [1], as opposed to very small specimens such as spheres used in conventional perturbation technique measurements. The propagation constants $\gamma_\pm = \alpha_\pm + j\beta_{g\pm}$ of right- and left-handed rotating HE_{11} waves in the ferrite-loaded waveguide are measured as function of a longitudinally applied static magnetic field. From these measurements the complex eigenvalues $\mathfrak{u}_\pm = \mu_\pm' - j\mu_\pm''$ of the permeability tensor of the ferrite can be evaluated. The relation between the propagation constants of the waves and the dimensions of the waveguide structure, the permeability tensor of the ferrite rod, and the material parameters of the surrounding medium (in this special case the surrounding medium was air with the assumed vacuum dielectric constant ϵ_0) is given by the characteristic equations of the problem [2]. Consequently the calculation of the four material parameters $\mu_\pm'$, $\mu_\pm''$ requires the solution of these characteristic equations forming a system of four transcendental equations. The structure of the computer program of this problem is shown in the simplified flow chart of Fig. 1. The source program consists of five parts—the main program and four subprograms. The main program includes the COMMON statement for common storage arrays, the READ and WRITE statements for the data input and output, and the CALL and EXTERNAL statements for the subprograms. In the subprograms FUNCTION *CFMUEP* and FUNCTION *CFMUEM* the characteristic equations for the right- and left-handed rotating H_{11} waves are programmed. The subprogram SUBROUTINE NEWTON is used to solve the system of the four transcendental equations by applying Newton's method [3]. By means of the subprogram SUBROUTINE COMBES, the Bessel functions of first and second kind with order 0 and 1 of complex argument occurring in the characteristic equations are calculated.

The computer run starts with reading in a data card with the actual values of the following parameters:

Manuscript received December 7, 1971; revised February 18, 1972.

For program listing, order document NAPS-01818 from ASIS National Auxiliary Publications Service, c/o CCM Information Corporation, 909 Third Avenue, New York, N. Y. 10022; remitting $2.00 per microfiche or $5.00 per photocopy.

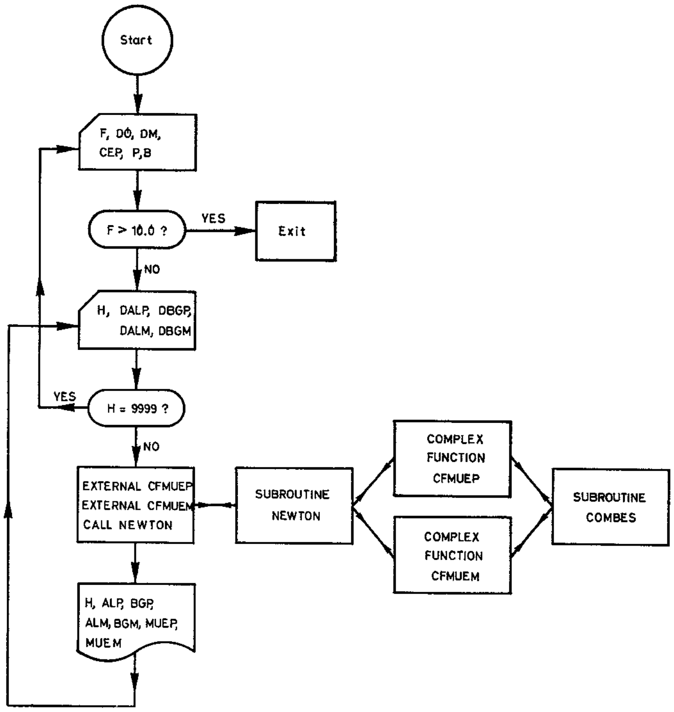

Fig. 1. Simplified flow chart of the computer program.

F	measuring frequency;
*D*0	inner diameter of the cylindrical waveguide;
DM	diameter of the ferrite rod;
CEP	complex permittivity of the ferrite;
P, *B*	6 sign word to mark the specimen.

A further READ statement reads in a second data card with the following input variables:

H	intensity of the static magnetic field;
DALP	difference between the attenuation constants of the right-handed rotating HE_{11} wave in the ferrite-loaded waveguide and of the H_{11} wave in the empty waveguide;
DBGP	difference between the phase constants of the right-handed rotating HE_{11} wave in the ferrite-loaded waveguide and of the H_{11} wave in the empty waveguide;
DALM, *DGBM*	corresponding to *DALP* and *DBGP* with left-handed rotating HE_{11} and H_{11} waves.

With the CALL statement for the subprogram NEWTON the complex eigenvalues $\mathfrak{u}_+$, $\mathfrak{u}_-$ (*MUEP*, *MUEM*) of the permeability tensor are calculated. The subprograms *CFMUEP* and *CFMUEM* containing the complex characteristic equations—inserted at the same time with EXTERNAL statements—call the subprogram COMBES for the calcula-